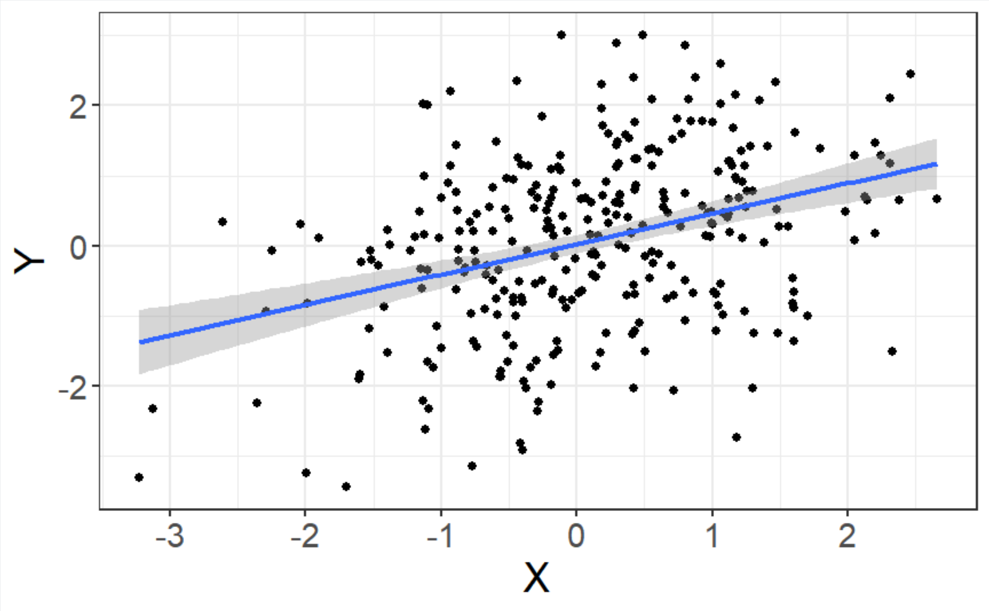
Multiple and logistic regression

2022/04/19

Linear regression - a (brief) recap

Linear regression

```
## `geom_smooth()` using formula 'y ~ x'
```



Our job is to figure out the mathematical relationship between our *predictor(s)* and our *outcome*.

$$Y = b_0 + b_1 X_i + \varepsilon_i$$

Linear regression

$$Y = b_0 + b_1 X_i + \varepsilon_i$$

Linear regression

$$Y = b_0 + b_1 X_i + \varepsilon_i$$

Y - The outcome - the dependent variable.

b_0 - The *intercept*. This is the value of Y when X = 0.

b_1 - The regression coefficients. This describes the steepness of the relationship between the outcome and *slope(s)*.

X_i - The predictors - our independent variables.

ε_i - The *random error* - variability in our dependent variable that is not explained by our predictors.

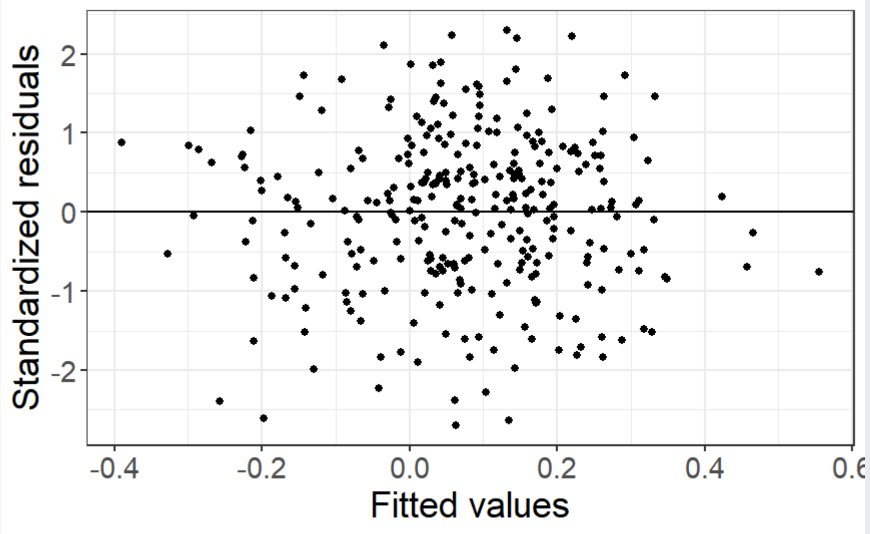
Regression assumptions

Regression assumptions

Linear regression has a number of assumptions:

- Normally distributed errors
- Homoscedasticity (of errors)
- Independence of errors
- Linearity
- No perfect multicollinearity

Normally distributed errors

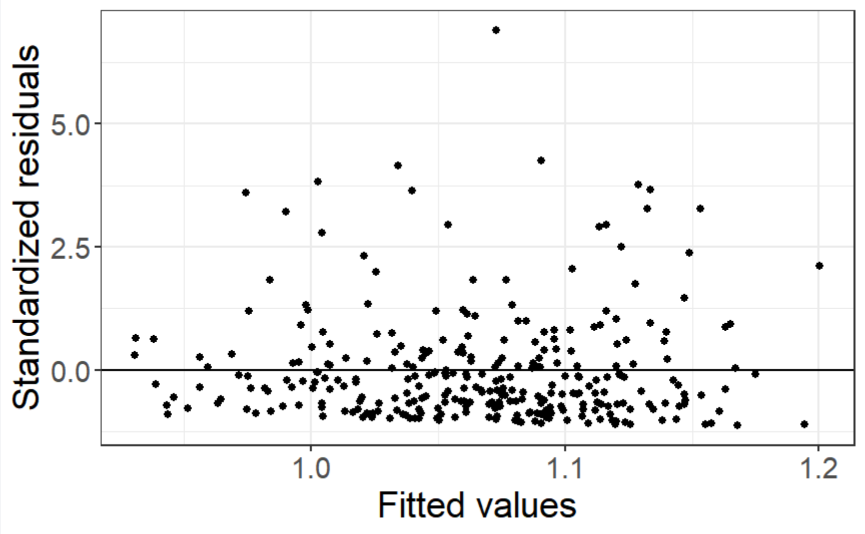


The *errors*, ε_i, are the variance left over after your model is fit.

An example like that on the left is what you want to see!

There is no clear pattern; the dots are evenly distributed around zero.

Skewed errors



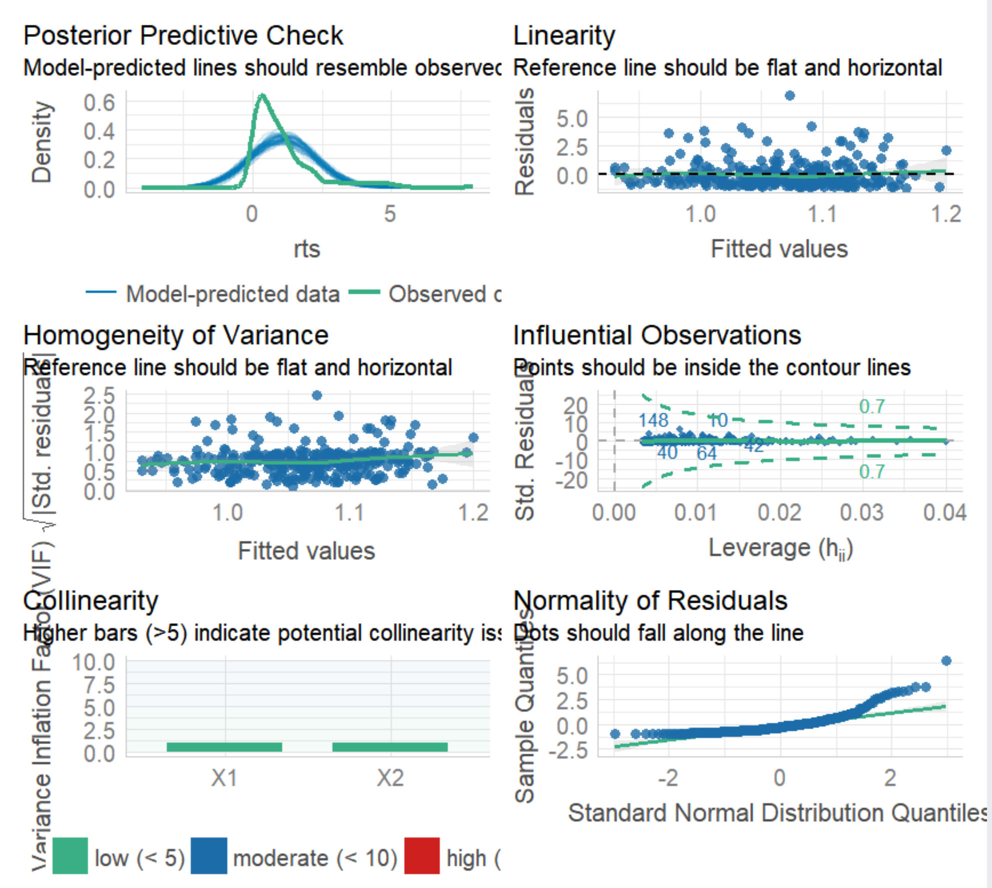
In contrast, the residuals on the left are skewed.

This most often happens with data that are *bounded*. For example, *reaction times* cannot be below zero; negative values are impossible.

Checking assumptions

The `performance` package has a very handy function called `check_model()`, which shows a variety of ways of checking the assumptions all at once.

```
library(performance)
check_model(test_skew)
```

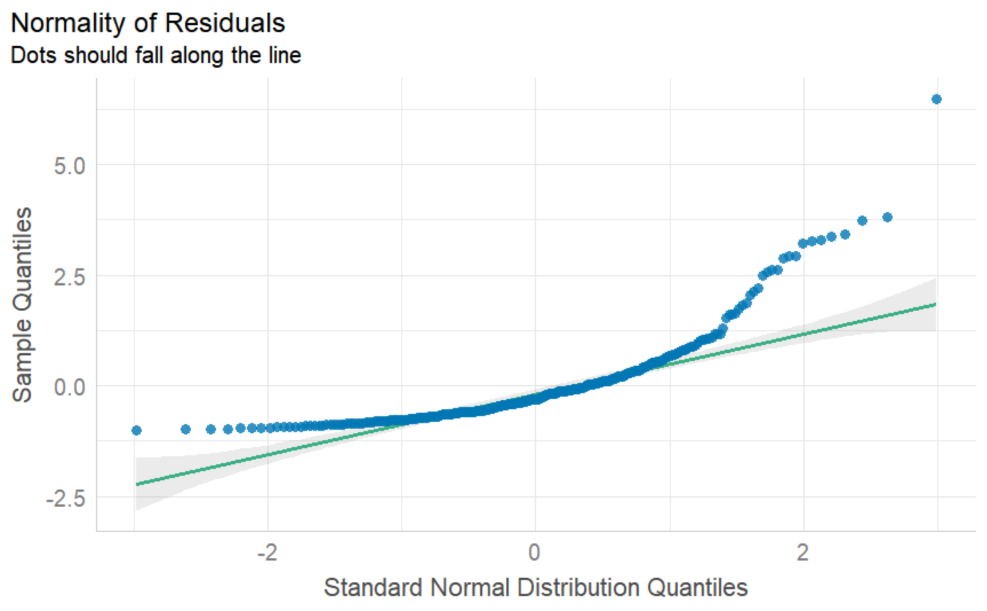


Checking assumptions

You can follow up suspicious looking plots with individual functions like `check_normality()`, which uses `shapiro.test()` to check the residuals and also provides nice plots.

Rely on the plots more than significance tests...

```
plot(check_normality(test_skew),
     type = "qq")
```



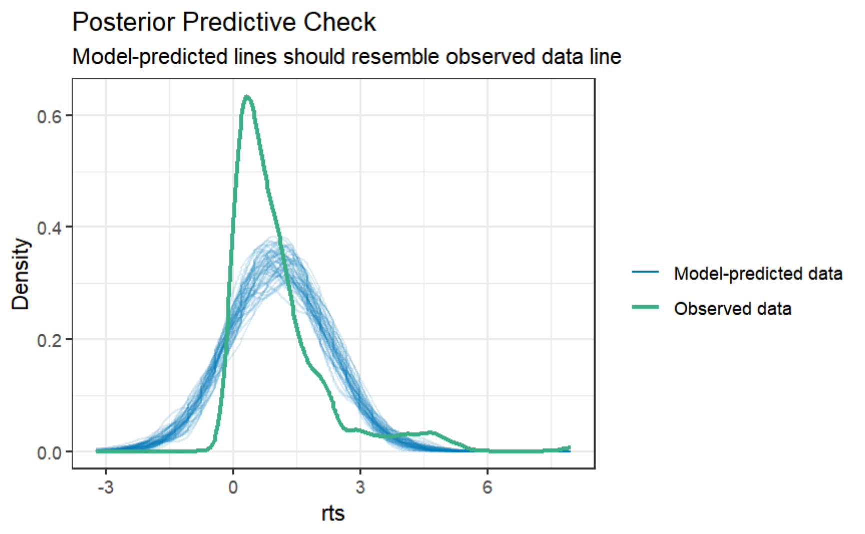
So, about violated assumptions? 😱

1) We can think about transformations 😱

2) We could consider *non-parametric stats* - things like `wilcox.test()`, `friedman.test()`, `kruskal.test()`, all of which are based on rank transformations and thus are really more like point 1 😱

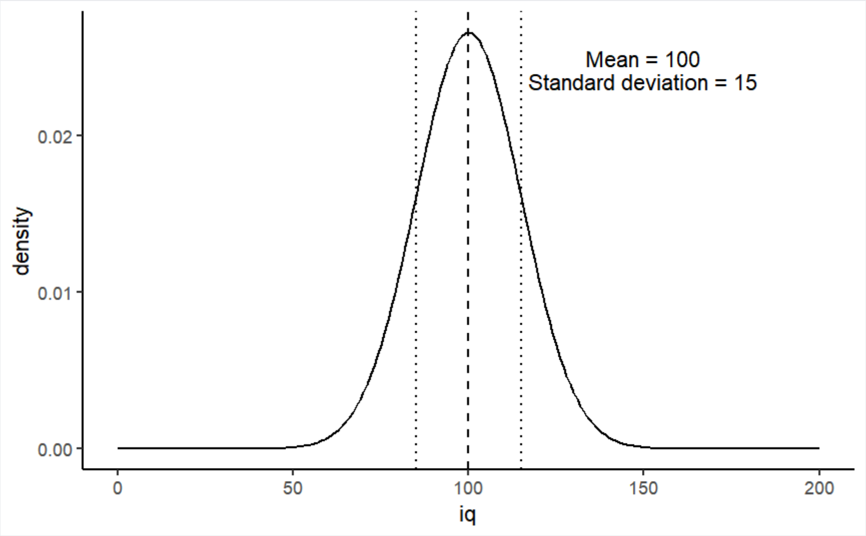
3) We should think about **why** the assumptions might be violated. Is this just part of how the data is *generated*? 🤔

```
## Warning: Maximum value of original data is not i
##   Model may not capture the variation of the dat
```



Generalized Linear Models

Distributional families



The *normal* distribution can also be called the *Gaussian* distribution.

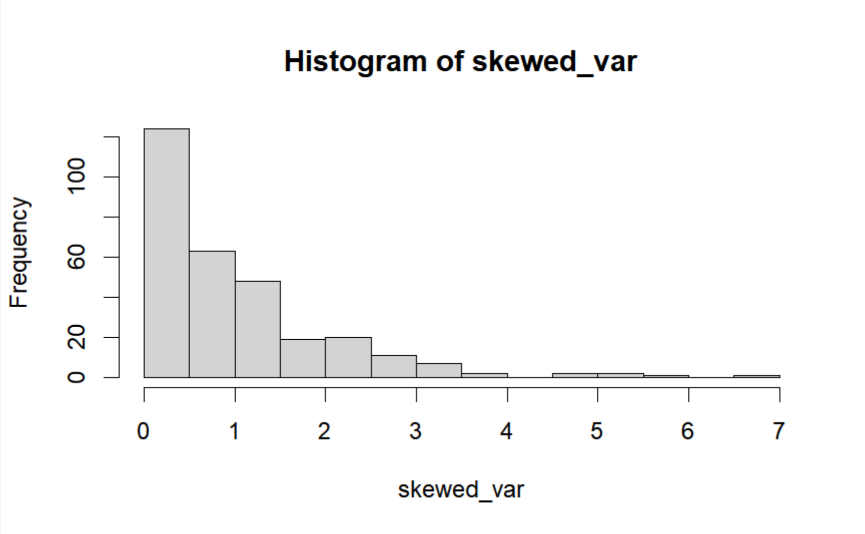
The linear regression models we've used so far assume a *Gaussian* distribution of errors.

Generalized linear models

A *Generalized Linear Model* - fit with `glm()` - allows you to specify what type of family of probability distributions the data are drawn from.

The data | With lm | With glm

```
skewed_var <- rgamma(300, 1)
hist(skewed_var)
```



Generalized linear models

A *Generalized Linear Model* - fit with `glm()` - allows you to specify what type of family of probability distributions the data are drawn from.

The data | **With lm** | With glm

```
lm(skewed_var ~ X1 + X2)
```

```
##
## Call:
## lm(formula = skewed_var ~ X1 + X2)
##
## Coefficients:
## (Intercept)           X1           X2
##     1.01292     -0.13856      0.02078
```

Generalized linear models

A *Generalized Linear Model* - fit with `glm()` - allows you to specify what type of family of probability distributions the data are drawn from.

The data | With lm | **With glm**

```
glm(skewed_var ~ X1 + X2, family = "gaussian")
```

```
##
## Call:  glm(formula = skewed_var ~ X1 + X2, family = "gaussian")
##
## Coefficients:
## (Intercept)           X1           X2
##     1.01292     -0.13856      0.02078
##
## Degrees of Freedom: 299 Total (i.e. Null);  297 Residual
## Null Deviance:       329.7
## Residual Deviance: 323.6     AIC: 882.1
```

Categorical outcome variables

Suppose you have a *discrete*, *categorical* outcome.

Examples of categorical outcomes:

- correct or incorrect answer
- person commits an offence or does not

Examples of counts:

- Number of items successfully recalled
- Number of offences committed

The binomial distribution

A coin only has two sides: heads or tails.

Assuming the coin is fair, the probability - P - that it lands on *heads* is *.5*. So the probability it lands on *tails* - $1 - P$ is also *.5*.

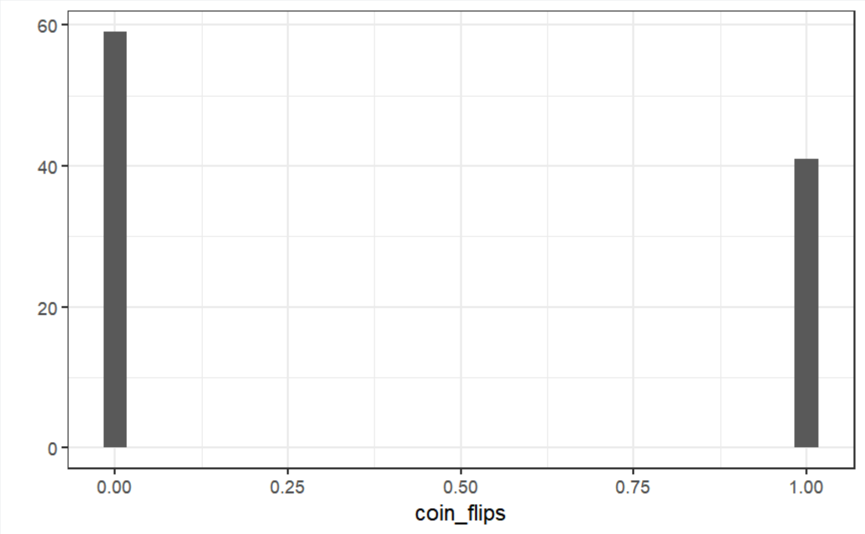
This type of variable is called a **Bernoulli random variable**.

If you toss the coin many times, the count of how many heads and how many tails occur is called a **binomial distribution**.

Binomial distribution

If we throw the coin 100 times, how many times do we get tails?

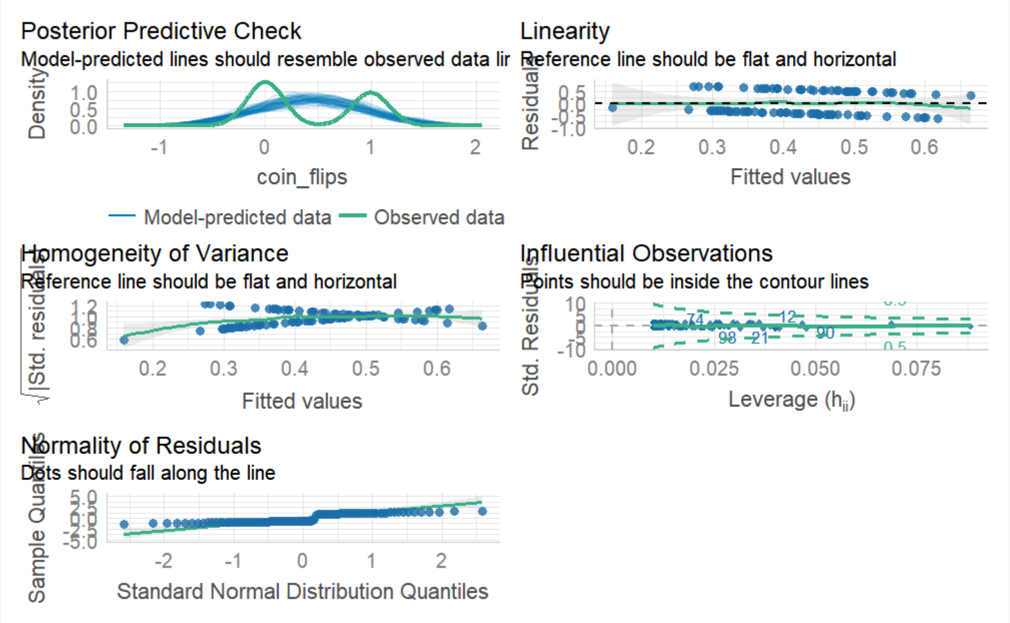
```
coin_flips <- rbinom(n = 100, size = 1, prob = 0.5)
qplot(coin_flips)
```



Binomial distribution

What happens if we try to model individual coin flips with `lm()`?

```
coin_flips <- rbinom(n = 100, size = 1, prob = 0.5)
x3 <- rnorm(100) # this is just a random variable for the purposes of demonstration!
check_model(lm(coin_flips ~ x3))
```



Logistic regression

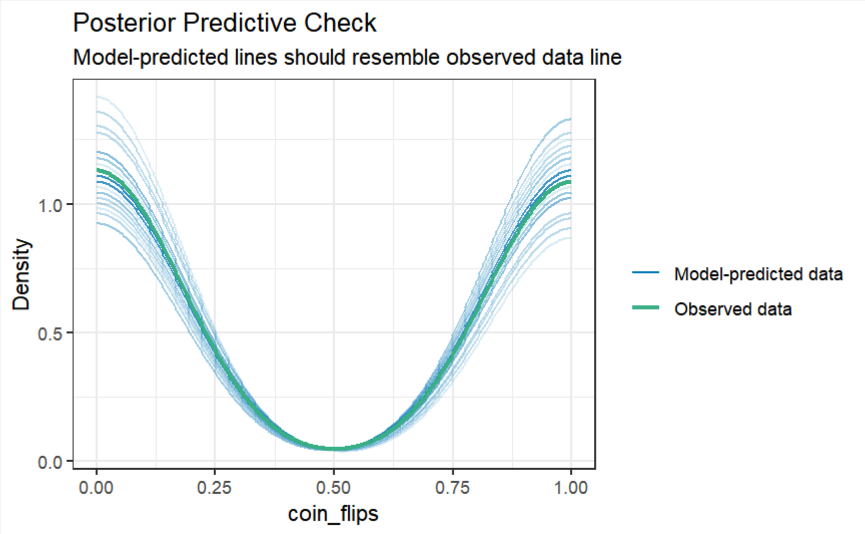
We get `glm()` to model a `binomial` distribution by specifying the *binomial* family.

```
coin_flips <- rbinom(n = 100, size = 1, prob = 0.5)
glm(coin_flips ~ 1,
    family = binomial(link = "logit"))
```

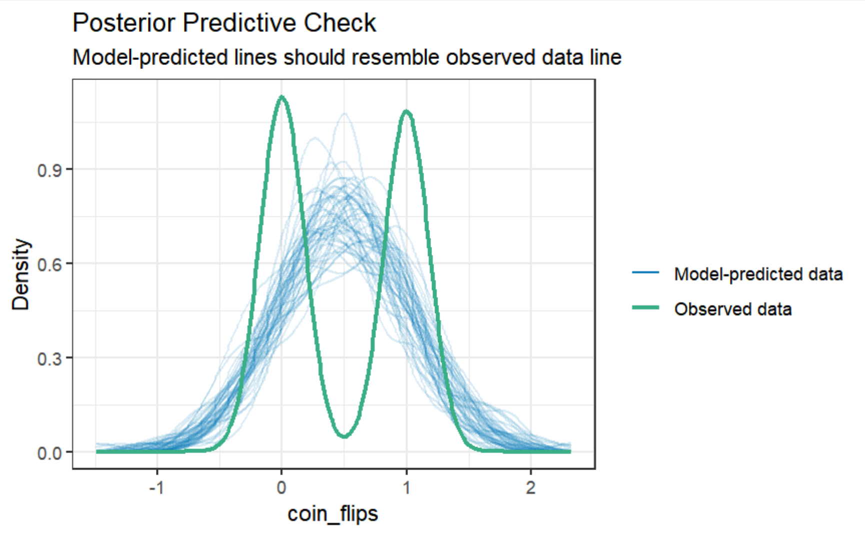
```
##
## Call:  glm(formula = coin_flips ~ 1, family = binomial(link = "logit"))
##
## Coefficients:
## (Intercept)
##    -0.04001
##
## Degrees of Freedom: 99 Total (i.e. Null);  99 Residual
## Null Deviance:        138.6
## Residual Deviance: 138.6     AIC: 140.6
```

Logistic regression

GLM with logit link 😎



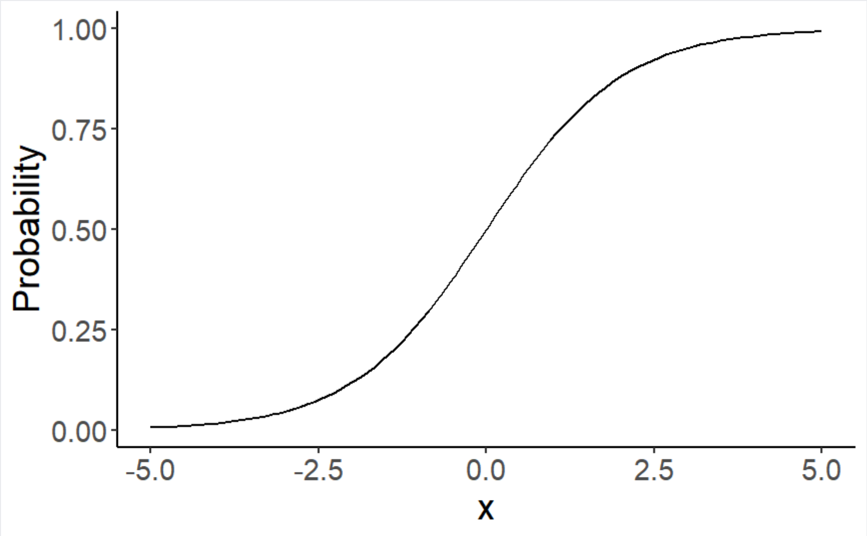
GLM with Gaussian link 😭



Logistic regression

The *logit* transformation is used to *link* our predictors to our discrete outcome variable.

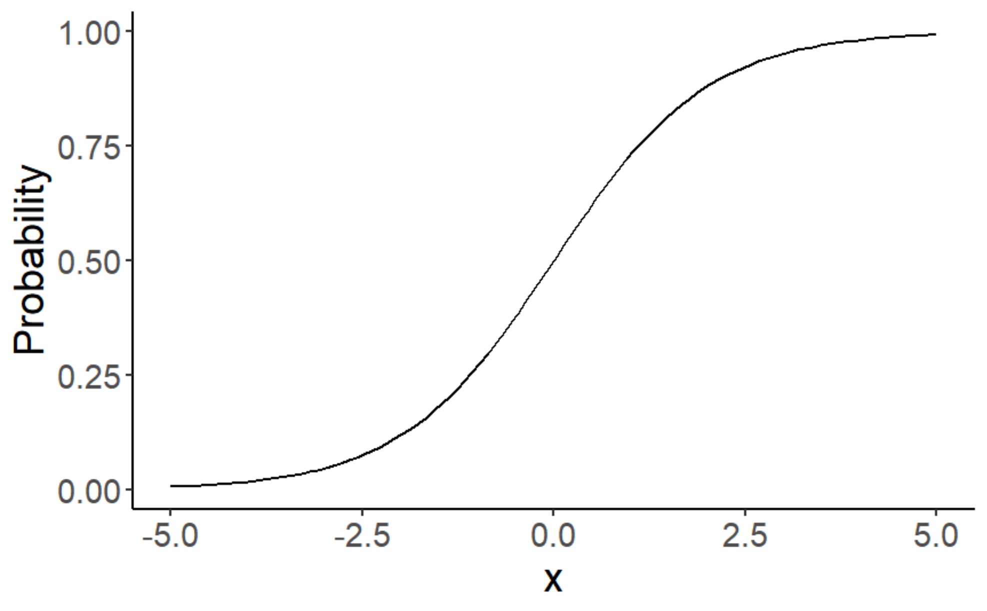
It helps us constrain the influence of our predictors to the range 0-1, and account for the change in *variance* with probability.



Logistic regression

As probabilities approach zero or one, the range of possible values *decreases*.

Thus, the influence of predictors on the *response scale* must also decrease as we reach one or zero.



Logistic regression

$$P(Y) = \frac{1}{1 + e^{-}(b_0 + b_1 X_1 + \varepsilon_i)}$$

$P(Y)$ - The *probability* of the outcome happening.

Logistic regression

$$P(Y) = \frac{1}{1 + e^{-}(b_0 + b_1 X_1 + \varepsilon_i)}$$

$P(Y)$ - The *probability* of the outcome happening.

$\frac{1}{1+e^{-}(...)}$ - The *log-odds* (logits) of the predictors.

Odds ratios and log odds

Odds are the ratio of one outcome versus the others. e.g. The odds of a randomly chosen day being a Friday are 1 to 6 (or 1/6 = .17)

Log odds are the *natural log* of the odds:

$$log(\frac{p}{1-p})$$

The coefficients we get out are *log-odds* - they can be hard to interpret on their own.

```
coef(glm(coin_flips ~ 1, family = binomial(link = "logit")))
```

```
## (Intercept)
## -0.04000533
```

Odds ratios and log odds

If we exponeniate them, we get the *odds ratios* back.

```
exp(coef(glm(coin_flips ~ 1, family = binomial(link = "logit"))))
```

```
## (Intercept)
##   0.9607843
```

So this one is roughly 1:1 heads and tails! But there's a nicer way to figure it out...

Taking penalties

Taking penalties

What's the probability that a particular penalty will be scored?

```
##   PSWQ Anxious Previous         Scored
## 1   18      21       56 Scored Penalty
## 2   17      32       35 Scored Penalty
## 3   16      34       35 Scored Penalty
## 4   14      40       15 Scored Penalty
## 5    5      24       47 Scored Penalty
## 6    1      15       67 Scored Penalty
```

- **PSWQ** = Penn State Worry Questionnaire
- **Anxiety** = State Anxiety
- **Previous** = Number of penalties scored previously

Taking penalties

```r
pens <- glm(Scored ~ PSWQ + Anxious + Previous,
            family = binomial(link = "logit"),
            data = penalties)
pens
```

```
##
## Call:  glm(formula = Scored ~ PSWQ + Anxious + Previous, family = binomial(link = "logit"),
##     data = penalties)
##
## Coefficients:
## (Intercept)         PSWQ      Anxious     Previous
##    -11.4926      -0.2514       0.2758       0.2026
##
## Degrees of Freedom: 74 Total (i.e. Null);  71 Residual
## Null Deviance:       103.6
## Residual Deviance: 47.42     AIC: 55.42
```

```
##
## Call:
## glm(formula = Scored ~ PSWQ + Anxious + Previous, family = binomial(link = "logit"),
##     data = penalties)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -2.31374  -0.35996   0.08334   0.53860   1.61380
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -11.49256   11.80175  -0.974  0.33016
## PSWQ         -0.25137    0.08401  -2.992  0.00277 **
## Anxious       0.27585    0.25259   1.092  0.27480
## Previous      0.20261    0.12932   1.567  0.11719
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 103.638  on 74  degrees of freedom
## Residual deviance:  47.416  on 71  degrees of freedom
## AIC: 55.416
##
## Number of Fisher Scoring iterations: 6
```

The response scale and the link scale

The model is fit on the *link* scale.

The coefficients returned by the GLM are in *logits*, or *log-odds*.

```
coef(pens)
```

```
## (Intercept)         PSWQ     Anxious    Previous
## -11.4925608  -0.2513693   0.2758489   0.2026082
```

How do we interpret them?

Converting logits to odds ratios

```
coef(pens)[2:4]
```

```
##        PSWQ    Anxious   Previous 
## -0.2513693  0.2758489  0.2026082
```

We can *exponentiate* the log-odds using the **exp()** function.

```
exp(coef(pens)[2:4])
```

```
##      PSWQ   Anxious  Previous 
## 0.7777351 1.3176488 1.2245925
```

Odds ratios

An odds ratio greater than 1 means that the odds of an outcome increase.

An odds ratio less than 1 means that the odds of an outcome decrease.

```
exp(coef(pens)[2:4])
```

```
##      PSWQ   Anxious  Previous
## 0.7777351 1.3176488 1.2245925
```

From this table, it looks like the odds of scoring a penalty decrease with increases in PSWQ but increase with increases in State Anxiety or Previous scoring rates.

The response scale

The *response* scale is even *more* intuitive. It makes predictions using the *original* units. For a binomial distribution, that's *probabilities*. We can generate probabilities using the **predict()** function.

```
penalties$prob <- predict(pens, type = "response")
head(penalties)
```

```
##   PSWQ Anxious Previous         Scored      prob
## 1   18      21       56 Scored Penalty 0.7542999
## 2   17      32       35 Scored Penalty 0.5380797
## 3   16      34       35 Scored Penalty 0.7222563
## 4   14      40       15 Scored Penalty 0.2811731
## 5    5      24       47 Scored Penalty 0.9675024
## 6    1      15       67 Scored Penalty 0.9974486
```

Model predictions

Note that these model predictions *don't need to use the original data*. Let's see how the probability of scoring changes as `PSWQ` increases.

Create new data | Make predictions | Plot predictions

```
new_dat <-
  tibble::tibble(PSWQ = seq(0, 30, by = 2),
                 Anxious = mean(penalties$Anxious),
                 Previous = mean(penalties$Previous))
```

Model predictions

Note that these model predictions *don't need to use the original data*. Let's see how the probability of scoring changes as `PSWQ` increases.

Create new data | **Make predictions** | Plot predictions

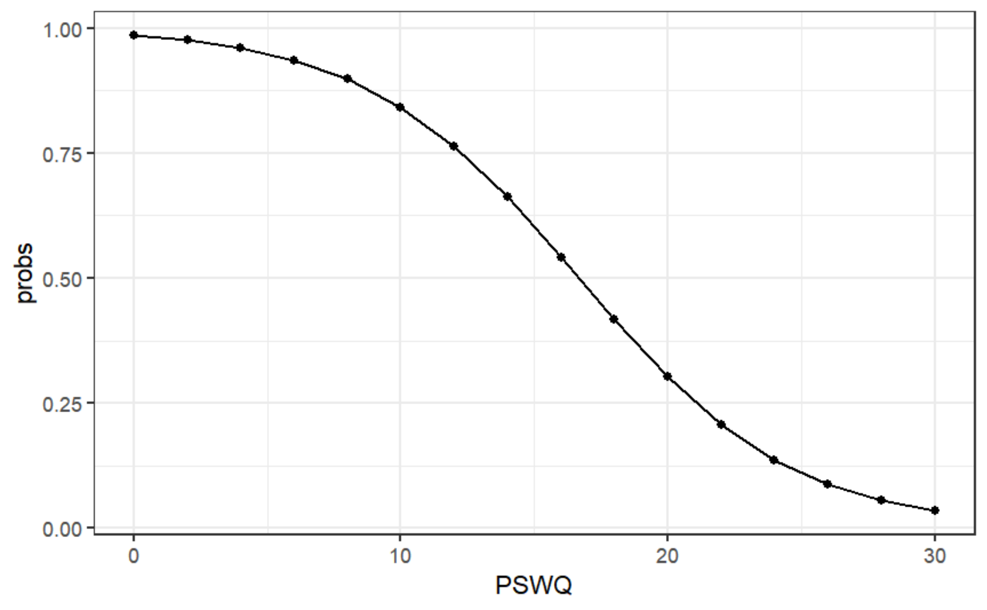
```
new_dat$probs <-
  predict(pens,
          newdata = new_dat,
          type = "response")
```

Model predictions

Note that these model predictions *don't need to use the original data*. Let's see how the probability of scoring changes as `PSWQ` increases.

Create new data | Make predictions | **Plot predictions**

```
ggplot(new_dat, aes(x = PSWQ, y = probs)) +
  geom_point() +
  geom_line()
```



Model predictions

Imagine you wanted to the probability of scoring for somebody with a PSWQ score of 7, an Anxious rating of 12, and a Previous scoring record of 34.

Make the data | Predict log-odds | Predict odds | Predict probabilities

```
new_dat <- tibble::tibble(PSWQ = 7,
                          Anxious = 22,
                          Previous = 34)
```

Model predictions

Imagine you wanted to the probability of scoring for somebody with a PSWQ score of 7, an Anxious rating of 12, and a Previous scoring record of 34.

Make the data | **Predict log-odds** | Predict odds | Predict probabilities

```
predict(pens, new_dat)
```

```
##          1
## -0.2947909
```

Model predictions

Imagine you wanted to the probability of scoring for somebody with a PSWQ score of 7, an Anxious rating of 12, and a Previous scoring record of 34.

Make the data | Predict log-odds | **Predict odds** | Predict probabilities

```
exp(predict(pens, new_dat))
```

```
##         1
## 0.7446873
```

Model predictions

Imagine you wanted to the probability of scoring for somebody with a PSWQ score of 7, an Anxious rating of 12, and a Previous scoring record of 34.

Make the data | Predict log-odds | Predict odds | **Predict probabilities**

```
predict(pens, new_dat, type = "response")
```

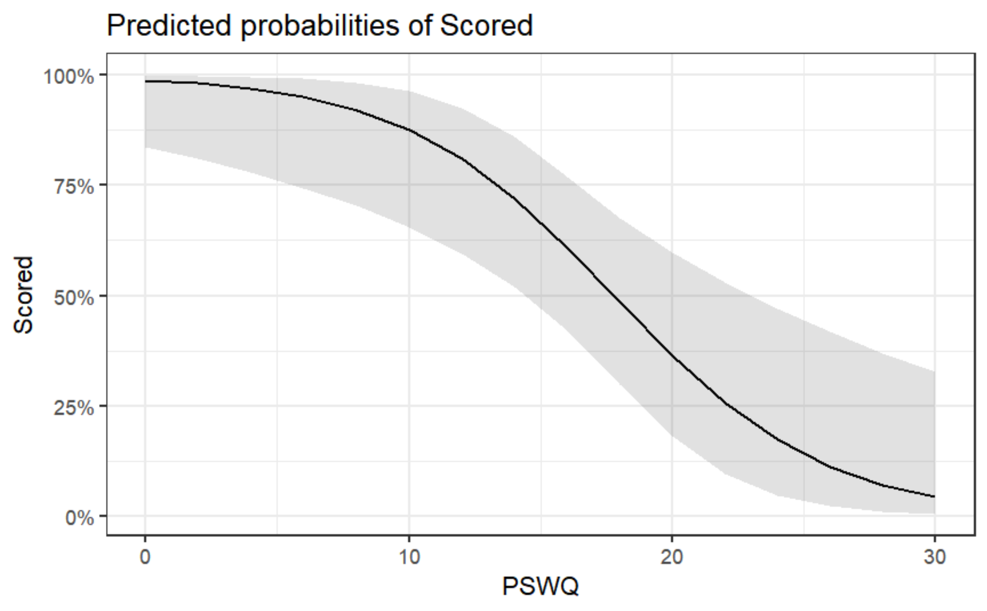
```
##         1
## 0.4268314
```

Plotting

The **sjPlot** package has some excellent built in plotting tools - try the **plot_model()** function.

```
library(sjPlot)
plot_model(pens,
           type = "pred",
           terms = "PSWQ")
```

```
## Data were 'prettified'. Consider using `terms=
```



Results tables

```
sjPlot::tab_model(pens)
```

	Scored		
Predictors	*Odds Ratios*	*CI*	*p*
(Intercept)	0.00	0.00 – 64258.63	0.330
PSWQ	0.78	0.64 – 0.90	**0.003**
Anxious	1.32	0.81 – 2.24	0.275
Previous	1.22	0.96 – 1.61	0.117
Observations	75		
R^2 Tjur	0.594		

The Titanic dataset

The Titanic dataset

The Titanic dataset

```
head(full_titanic)
```

```
## # A tibble: 6 x 12
##   PassengerId Survived Pclass Name        Sex     Age SibSp Parch Ticket  Fare Cabin
##         <dbl>    <dbl>  <dbl> <chr>       <chr> <dbl> <dbl> <dbl> <chr>  <dbl> <chr>
## 1           1        0      3 Braund, M~ male     22     1     0 A/5 2~  7.25 <NA>
## 2           2        1      1 Cumings, ~ fema~    38     1     0 PC 17~ 71.3  C85
## 3           3        1      3 Heikkinen~ fema~    26     0     0 STON/~  7.92 <NA>
## 4           4        1      1 Futrelle,~ fema~    35     1     0 113803 53.1  C123
## 5           5        0      3 Allen, Mr~ male     35     0     0 373450  8.05 <NA>
## 6           6        0      3 Moran, Mr~ male     NA     0     0 330877  8.46 <NA>
## # ... with 1 more variable: Embarked <chr>
```

Downloaded from Kaggle

The Titanic dataset

The Titanic dataset

```r
full_titanic %>%
  group_by(Survived,
           Sex) %>%
  count()
```

```
## # A tibble: 4 x 3
## # Groups:   Survived, Sex [4]
##   Survived Sex        n
##      <dbl> <chr>  <int>
## 1        0 female    81
## 2        0 male     468
## 3        1 female   233
## 4        1 male     109
```

```r
full_titanic %>%
  group_by(Sex) %>%
  summarise(p = mean(Survived),
            Y = sum(Survived),
            N = n())
```

```
## # A tibble: 2 x 4
##   Sex        p     Y     N
##   <chr>  <dbl> <dbl> <int>
## 1 female 0.742   233   314
## 2 male   0.189   109   577
```

The Titanic dataset

```
##
## Call:
## glm(formula = Survived ~ Age + Pclass, family = binomial(), data = full_titanic)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1524  -0.8466  -0.6083   1.0031   2.3929
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  2.296012   0.317629   7.229 4.88e-13 ***
## Age         -0.041755   0.006736  -6.198 5.70e-10 ***
## Pclass2     -1.137533   0.237578  -4.788 1.68e-06 ***
## Pclass3     -2.469561   0.240182 -10.282  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 964.52  on 713  degrees of freedom
## Residual deviance: 827.16  on 710  degrees of freedom
```

The Titanic dataset

```r
library(emmeans)
emmeans(age_class,
        ~Age|Pclass,
        type = "response")
```

```
## Pclass = 1:
##   Age  prob     SE  df asymp.LCL asymp.UCL
##  29.7 0.742 0.0339 Inf     0.670     0.803
##
## Pclass = 2:
##   Age  prob     SE  df asymp.LCL asymp.UCL
##  29.7 0.480 0.0394 Inf     0.403     0.557
##
## Pclass = 3:
##   Age  prob     SE  df asymp.LCL asymp.UCL
##  29.7 0.196 0.0216 Inf     0.157     0.241
##
## Confidence level used: 0.95
## Intervals are back-transformed from the logit scale
```

Some final notes on Generalized Linear Models

Today has focussed on **logistic** regression with *binomial* distributions.

But Generalized Linear Models can be expanded to deal with many different types of outcome variable!

e.g. *Counts* follow a Poisson distribution - use `family = "poisson"`

Ordinal variables (e.g. Likert scale) can be modelled using *cumulative logit* models (using the **ordinal** or **brms** packages).

Suggested reading for categorical ordinal regression

Liddell & Kruschke (2018). Analyzing ordinal data with metric models: What could possibly go Wrong?

Buerkner & Vuorre (2018). Ordinal Regression Models in Psychology: A Tutorial